On the optimal tuning of individual pitch control for horizontal-axis wind turbines

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Abstract
Individual pitch control has proved capable of reducing loads on the blades and shaft of a horizontal-axis wind turbine, at the price of a potentially substantial increase in actuator activity. With accurate tuning of the control parameters, the overall performance can be satisfactorily balanced, but a manual tuning procedure often proves unbearably costly and the solution found may be far from optimal. This study tries to explore the feasibility of an optimal approach to the problem of tuning of individual pitch control, by mapping reasonable cost functions with respect to some parameters of interest in control design. The analysis is carried out by considering a possible individual pitch control implementation, tested in a virtual environment on a realistic testbed. The merit functions are also analyzed visually, in order to easily understand the effects of the various design parameters on the positions and quality of the respective optima. In a later stage, results of complete optimization runs are presented, and the approach is critically discussed.

Keywords
Individual pitch control, control parameters, optimization, optimal tuning, horizontal-axis wind turbine

Introduction
In recent years, much attention has been devoted to the reduction of fatigue loads by means of individual pitch control, as demonstrated by the rich literature dedicated to the subject (see, for instance, Bossanyi, 2003b,a, 2004, 2005; Bottasso et al., 2013; Geyler and Caselitz, 2007, 2008; Leithead et al., 2009; Stol, 2003; Stol and Balas, 2003; Van Engelen, 2006; Van Engelen and Kanev, 2009). Notwithstanding extensive analysis of the topic, and the relevant advantage that this control can provide on loads, individual pitch control has not generally been adopted by the industry, owing to the potentially excessive workload imposed on pitch actuators, reflected in an increased value of the actuator duty cycle with respect to that obtained when operating with a reference collective control.

The need to limit the action of pitch actuators without relinquishing the mitigating effect on loads provided by individual pitch control can be dealt with by proper tuning of the parameters of the individual pitch control law, mainly the proportional and integral gains for a model-free approach (Bossanyi, 2003b) or an array of control and load weightings, in the case of model-based control laws (Bottasso et al., 2013). Control tuning is the result of a trade-off analysis and is usually carried out by means of a time-consuming manual procedure. Bottasso et al. (2014) restrict the use of individual pitch control to a limited range of wind speeds, selected to maximize the control effectiveness while minimizing actuator use. Yet also in that case, the selection of the activation range is completed by a manual trial and error procedure.

The issue represented by the need to contain the workload imposed on actuators while effectively reducing loads, without going through a lengthy manual tuning procedure, can be addressed by selecting an optimal approach to determining the tuning parameters. A similar approach has been successfully tried for setting the parameters of a control law for turbine trimming (Bottasso et al., 2006).

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This study explores the effect of some relevant tuning parameters for a realistic individual pitch control law on a selection of possible merit functions. The merit functions will be analyzed by considering test scenarios of increasing complexity. An approach by sub-problems is proposed, in which the optimum of a merit function with respect to at most two optimization parameters is searched for, to enable visual mapping of the merit function. This favors an easier observation of the effects of changing the values of the parameters of interest on the selected merit function and on its optimum.

It should be emphasized that the main focus of the work lies not in optimizing the control parameters, but mainly in understanding what is a good way to set up the optimization problem based on such parameters. With this in mind, part of the paper is devoted to an analysis of the general behavior of the proposed merit functions with respect to changes in the balance of the quantities accounted for in their definition, i.e. measures of loads and actuator duty cycle.

The outcome of this study is primarily to establish whether an optimal approach to control tuning is feasible and can be of help to engineers, providing a way to almost automatically select the values that control parameters should assume to optimize a comprehensive measure of performance, and secondarily to provide some elements for determining what control design parameters and what testing scenarios should be taken into account in an optimal approach to the problem of control tuning. Finally, in case no optimization tool is available in the tuning phase, this study provides indications on how to reasonably tune the considered parameters.

The paper is organized as follows. The next section briefly presents the individual pitch control architecture chosen for the analysis. The central section is devoted to the presentation of the considered scenarios of analysis. In the results section, diagrams of the cost functions obtained in several different cases considered for this study are presented and discussed and the results of some full optimization runs are shown, supporting what can be concluded from the preliminary analysis of the cost functions. Finally, a summary of the results and some critical considerations are presented.

**Individual pitch control law**

As recalled in the introduction, there are a number of ways to implement an individual pitch control, witnessed by the wide extent of literature on the topic. A model-free individual pitch control, similar to that proposed by Bossanyi (2005), was selected for this analysis, owing to its greater simplicity with respect to other implementations. The adopted control scheme differs from that presented in the works of Bossanyi in its use of both out-of-plane and in-plane blade load signals instead of out-of-plane blade load signals only (Riboldi, 2012). The main features of the adopted implementation will be outlined in this section.

The individual pitch control law takes as input the out-of-plane and in-plane bending moments at the root of the blades, \( M_{\text{OoP}} = \{M_{\text{OoP,1}}, M_{\text{OoP,2}}, M_{\text{OoP,3}}\}^T \) and \( M_{\text{InP}} = \{M_{\text{InP,1}}, M_{\text{InP,2}}, M_{\text{InP,3}}\}^T \), respectively, for a three-bladed turbine. These are transformed by means of the 1-per-rev (1P) Coleman transform into two fixed-frame equivalent loads (Van Engelen, 2006)

\[
M_{\text{d-q}} = \begin{bmatrix} M_d \\ M_q \end{bmatrix} = C_{\text{OoP}}(\Psi)M_{\text{OoP}} + C_{\text{InP}}(\Psi)M_{\text{InP}}
\]

where \( \Psi \) represents the azimuth of the rotor, measured as that of the first blade. The two transform matrices in equation (1) are defined as

\[
C_{\text{OoP}}(\Psi) = \begin{bmatrix} \cos(\Psi) & \cos(\Psi + \frac{2\pi}{3}) & \cos(\Psi + \frac{4\pi}{3}) \\ \sin(\Psi) & \sin(\Psi + \frac{2\pi}{3}) & \sin(\Psi + \frac{4\pi}{3}) \end{bmatrix}
\]

\[
C_{\text{InP}}(\Psi) = \begin{bmatrix} -\sin(\Psi) & -\sin(\Psi + \frac{2\pi}{3}) & -\sin(\Psi + \frac{4\pi}{3}) \\ \cos(\Psi) & \cos(\Psi + \frac{2\pi}{3}) & \cos(\Psi + \frac{4\pi}{3}) \end{bmatrix}
\]

where the angle \( \gamma \) is the cone angle of the rotor, considered positive when bending the blades towards the upwind direction.

The \( M_{\text{d-q}} \) signals in equation (1) are subsequently fed to two decoupled proportional-integral loops, producing two control signals in the fixed frame.
\[
\begin{bmatrix}
\beta_3 \\
\beta_4
\end{bmatrix} = k_p \mathbf{M}_{d-q} + k_i \int_0^t \mathbf{M}_{d-q} \, dt
\]

where \( k_p \) and \( k_i \) are 2-by-2 diagonal gain matrices.

The resulting \( \beta_3 \) and \( \beta_4 \) control signals are transformed back into time the domain using the inverse Coleman transform, yielding

\[
\begin{bmatrix}
\Delta \beta_3 \\
\Delta \beta_2 \\
\Delta \beta_1
\end{bmatrix} = C^* (\Psi) \begin{bmatrix}
\beta_3 \\
\beta_4
\end{bmatrix}
\]

where the matrix representing the inverse transformation is

\[
C^* (\Psi) = \begin{bmatrix}
\cos (\Psi) & \sin (\Psi) \\
\cos (\Psi + \frac{2\pi}{3}) & \sin (\Psi + \frac{2\pi}{3}) \\
\cos (\Psi + \frac{4\pi}{3}) & \sin (\Psi + \frac{4\pi}{3})
\end{bmatrix}
\]

The \( \Delta \beta_i, i = 1, 2, 3 \) pitch control signals obtained from equation (4), which sum up to zero at every time step by construction, can be added to the pitch value \( \beta_c \) coming from a collective controller used for trimming the machine to the correct rotor speed and power output in a multilayer fashion, as explained in Riboldi (2012) and Bottasso et al. (2013), yielding

\[
\beta_i = \beta_i + \Delta \beta_i, \quad i = 1, 2, 3
\]

As suggested in Van Engelen (2006), to avoid the generation of spurious control signals at frequencies higher than the 1P frequency, the load signals are low-pass filtered so that only frequencies up to the 1P frequency are fed to the Coleman transform.

In principle, there are four gain coefficients for this individual pitch control law (two for each matrix \( k_p \) and \( k_i \)); however, in practice, the controller usually produces satisfying results when considering the same gains for the two decoupled loops (Bottasso et al., 2013, 2014; Riboldi, 2012). This simplifies tuning, allowing the number of scalar control design parameters to be set at two for this control law, thus assuming a gain structure like \( k_p = k_p \cdot I \) and \( k_i = k_i \cdot I \), where \( I \) is a 2-by-2 identity matrix.

To limit the increase in actuator duty cycle due to individual pitch control activity, a logic for conditional activation of individual pitch control has been implemented, where the triggering variable is the speed of the air stream \( U(t) \) as measured by an anemometer on top of the tower (Bottasso et al., 2014). The number of activation ranges, i.e. intervals of wind speed values for which the individual pitch control is active, has been set to two, where the first can be centered at the rated wind speed \( U_r \) and the second at the cut-out \( U_o \). The individual pitch control is activated when the wind speed \( U(t) \) measured by the anemometer is such that

\[
U_{r_o} - \frac{R_o}{2} \leq U(t) \leq U_{r_o} + \frac{R_o}{2} \quad \text{or} \quad U_{o_o} - \frac{R_o}{2} \leq U(t) \leq U_{o_o} + \frac{R_o}{2}
\]

Owing to their predictably relevant impact on actuator duty cycle and load performance indices, the amplitudes \( R_o \) and \( R_o \) of the two activation ranges have been investigated as tuning parameters for the control law, besides the gains \( k_p \) and \( k_i \). The effect of such parameters will be presented in detail in the results section.

The frequent activation and deactivation of individual pitch control consequent to a time-varying wind speed, as is typical in turbulence, may generate frequent and unwanted local peak loads, in turn increasing fatigue. To minimize this effect, two countermeasures have been adopted. Firstly, a smooth control activation and deactivation procedure has been implemented. According to such a procedure, the controller gains are brought from zero to their design values and vice versa for activation and deactivation, respectively, following a time-linear law. The switching procedure is triggered
whenever the wind speed enters or leaves an operational range. Secondly, to avoid frequent activations or deactivations, especially when working close to one of the boundaries of the operational ranges, the wind speed signal coming from the sensor at the top of the tower is low-pass filtered with a moving-average filter with an appropriate averaging window of the order of some seconds. The duration of the activation and deactivation procedures can be set according to necessity and can be of the order of a few seconds.

Finally, practical experience suggests that the action of individual pitch control should be limited when the machine is operating in the region close to the rated wind speed, where the trimmer typically passes from controlling through torque only (partial power regions II and III), owing to the blades working in the proximity of a hard minimum pitch limit, to a condition where the pitch is free to change and both pitch and torque control are usually active for trimming (full power region III). To avoid interference in the basic trimming action from the superimposed individual pitch control component (Bottasso et al., 2013), the action of the latter is depressed by multiplying the value of the gains $k_p$ and $k_i$ by a factor $\zeta < 1$ when the machine is operated in such critical regions.

**Build-up of merit functions**

The action of individual pitch control usually produces visible reductions in the $1P$ frequency components of the target loads, i.e. the out-of-plane and in-plane moments $M_{\text{OoP}}$ and $M_{\text{InP}}$ at the root of the blades, with respect to what is obtained when only the trimmer is controlling the machine (Bossanyi, 2003b,a). As a matter of fact, thanks to the relationship between those blade loads and the $M_{d,q}$ moments expressed by the Coleman transform (equation (1)), a reduction in the constant component of the nodding and yawing bending moments measured on a cross-section of the shaft in a fixed system $M_s = \{M_{\text{vq}}, M_{\text{vd}}\}^T$ (Bossanyi, 2004) is reflected in a reduction in the $1P$ amplitude of blade loads measured in the rotating system. The moments $M_s$ are not defined exactly as $M_{d,q}$, at least because the actual nodding and yawing moment are also affected by the moment arising from the total shear forces at the roots of the blades. However, the values in $M_{d,q}$ and $M_s$ are related and usually proportional (Riboldi, 2012).

**Ideal constant wind conditions**

The theory of individual pitch control is mainly developed under ideal, strictly time-periodic conditions (Bossanyi, 2004; Bottasso et al., 2013; Van Engelen and Kanev, 2009). Therefore, a first performance assessment of individual pitch control laws should be carried out in ideal constant wind conditions, generating fully periodic time signals in the machine response, owing to shear and gravity effects. Advantages in the load values are usually clearly visible in such a scenario, in the form of a reduction of the peak-to-peak amplitude in the time histories of the blade moments, dominated by the $1P$ frequency (Bottasso et al., 2013; Riboldi, 2012). The peak-to-peak amplitude at the $1P$ frequency, $\Delta_{\text{pp}}$, can be obtained for a generic signal expressed as a function of the azimuth $s = s(\psi)$. It can be computed by demodulating the signal at the $1P$ frequency (Riboldi, 2012). Considering a demodulation over one rotor revolution, the amplitude of the $1P$ component in the signal $s$ can be defined in analytical terms as

\[
A_{1P} = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} d\psi s(\psi) \sin(\omega_1 \psi) d\psi
\]

\[
A_{1P} = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} d\psi s(\psi) \cos(\omega_1 \psi) d\psi
\]

\[
A_{1P} = \sqrt{A_{1P, \text{sin}}^2 + A_{1P, \text{cos}}^2}
\]

so that the peak-to-peak difference at the $1P$ frequency is $\Delta_{\text{pp}} = 2A_{1P}$. Considering all three blades, the average of the peak-to-peak amplitude can be written as

\[
\tilde{\Delta}_{\text{pp}} = \frac{1}{3} \sum_{i=1}^{3} \Delta_{\text{pp}}(M_{\text{OoP,i}})
\]

\[
\tilde{\Delta}_{\text{pp}} = \frac{1}{3} \sum_{i=1}^{3} \Delta_{\text{pp}}(M_{\text{InP,i}})
\]

where the symbol $\tilde{()}$ denotes an average over the three blades.
It is worth noting that the amplitude corresponding to an assigned harmonic component can also be computed with the azimuth-based method highlighted in equation (8) when the signal is not periodic in time. However, the amplitude corresponding to a certain frequency in the generic signal would clearly change in time in non-ideal conditions, i.e. when the wind speed is changing in time, as in turbulence. Hence, the peak-to-peak difference is of particular relevance only in ideal constant wind conditions, yielding a strictly periodic behavior of all load measures of interest.

As with the blades, the constant components of the nodding and yawing moments on a cross-section of the shaft are also expected to be reduced by individual pitch control in constant wind. They can be defined as

$$M_{\text{nod}} = \frac{1}{t} \int_{0}^{t} M_{\text{nod}}(\tau) d\tau$$
$$M_{\text{yaw}} = \frac{1}{t} \int_{0}^{t} M_{\text{yaw}}(\tau) d\tau$$

(10)

where the symbol $\overline{()}$ denotes an average in time.

As previously stated, the advantage on selected loads comes at the price of an increased actuator duty cycle, reflecting a more conspicuous motion of pitch actuators. The definition of $\text{ADC}_i$ adopted here for each blade is based on pitch rate (Riboldi, 2012; Stol, 2003), yielding

$$\text{ADC}_i = \frac{1}{t} \int_{0}^{t} |\dot{\beta}_i(\tau)| d\tau, \quad i = 1, 2, 3$$

(11)

where $(\cdot)$ represents the time derivative. The average actuator duty cycle can be defined as $\overline{\text{ADC}} = \frac{1}{3} \sum_{i=1}^{3} \text{ADC}_i$.

A merit function enabling simultaneous monitoring of both the expected increase in load performance and actuator motion in ideal constant wind conditions may be based on average peak-to-peak blade load, shaft cross-sectional bending average and average actuator duty cycle. To cover all operational regions, several constant wind turbine runs at a number $N_{U_i,e}$ of wind intensities $U_{0,i}, i = 1, \ldots, N_{U_i,e}$ should be considered between cut-in and cut-out. For a proper scaling of a yet-to-be-defined merit function, the normalized values of these quantities should be preferred over the corresponding dimensional definition. Furthermore, to ensure that the merit function is definite positive, it is necessary to take the absolute value of all quantities of interest. Therefore, for the quantities defined in equations (9) to (11), the corresponding positive, normalized values can be defined as

$$\rho\left(\hat{\text{IPC}}_{\text{pp}} M_{\text{Out}}\right) = \frac{\hat{\text{IPC}}_{\text{pp}} M_{\text{Out}}}{\hat{\text{IPC}}_{\text{ref}} M_{\text{Out}}}
\rho\left(\hat{\text{IPC}}_{\text{hp}} M_{\text{Out}}\right) = \frac{\hat{\text{IPC}}_{\text{hp}} M_{\text{Out}}}{\hat{\text{IPC}}_{\text{ref}} M_{\text{Out}}}
\rho\left(\overline{M}_{\text{nod}}\right) = \frac{\overline{M}_{\text{nod}}}{\overline{M}_{\text{nod}}_{\text{ref}}}
\rho\left(\overline{M}_{\text{yaw}}\right) = \frac{\overline{M}_{\text{yaw}}}{\overline{M}_{\text{yaw}}_{\text{ref}}}
\rho\left(\overline{\text{ADC}}\right) = \frac{\overline{\text{ADC}}}{\overline{\text{ADC}}_{\text{ref}}_{\text{ref}}}

(12)

where subscript $[\cdot]_{\text{IPC}}$ indicates results obtained with individual pitch control activated, whereas $[\cdot]_{\text{ref}}$ indicates the reference condition where individual pitch control is not active and only the trimmer is working to control the machine. Divisions are performed element by element on arrays, i.e. on the first line of equation (12). Note that $\overline{\text{ADC}}$ is normalized with respect to $\overline{\text{ADC}}_{\text{ref}}$, defined as

$$\overline{\text{ADC}}_{\text{ref}} = \begin{cases} \text{[ADC]}_{\text{ref}} & \text{if } \text{[ADC]}_{\text{ref}} \neq 0 \\ \sigma > 0 & \text{if } \text{[ADC]}_{\text{ref}} = 0 \end{cases}$$

(13)

This is to avoid division by zero in the possible case of a null reference $\overline{\text{ADC}}_{\text{ref}}$, which can be encountered for a very low wind speed $U_0$ in constant wind conditions when the trimmer is operating alone, usually at pitch for maximum power.
coefficient (partial power region II), owing to the usually slight collective pitch activity in this region (Bottasso et al., 2012). The value of $\sigma$ may be set close to a typical value of $ADC$ encountered with individual pitch control deactivated for the lower portion of full power region III, i.e. for those low wind speed values above the rated value for which pitch control is free.

A well tuned individual pitch control law working in a constant wind scenario should decrease all load indices in equation (12), except $\rho(ADC)$. Therefore, a possible synthetic measure of the overall performance of the individual pitch controller can take the form of the following merit function

$$ J_c = \frac{1}{N_u c} \sum_{j=1}^{N_u c} \left( g_j \rho(\Delta p \rho_{mp}\rho_{mp}) + h_j \rho(\Delta p \rho_{hp}\rho_{hp}) + I_j \rho(\Delta p \rho_{nod}\rho_{nod}) + m_j \rho(\Delta p \rho_{yaw}\rho_{yaw}) + a_j \rho(\Delta p \rho_{ADC}\rho_{ADC}) \right) $$

(14)

where $g_j$, $h_j$, $I_j$, $m_j$ and $a_j$ are positive scalar weightings.

Note that all components in $J_c$ except $\rho(ADC)$ are expected to be less than unity for increasing reasonable values of the gain multipliers $k_{pC}$, $k_{iC}$ and of the operational ranges $R_{pC}$ and $R_{oC}$ when individual pitch control is working properly. Furthermore, the components of the merit function are all positive by definition. Therefore, $J_c$ is definite positive, and the load and control indices in it tend to be lowered and raised, respectively, by a more intense control action, hence possibly producing a minimum for $J_c$.

**Turbulent wind conditions**

When moving to a turbulent wind scenario, the mitigating effect of individual pitch control on loads cannot easily be quantified by studying the performance indices adopted for a constant wind scenario. In particular, in the new condition, the change in the damage equivalent load computed from turbulent time histories is more representative than the 1P peak-to-peak difference, owing to the non-periodic time behavior of the load signals in turbulence.

The damage equivalent load can be computed using a standard rainflow technique, obtaining one load value equivalent to one time history (Bottasso et al., 2013; Riboldi, 2012). The damage equivalent load can be computed for both blade loads and shaft loads, and it will be indicated here by the operator $\Lambda = \text{DEL}(\cdot)$. Consequently, load values relevant for the analysis in turbulence are $\Lambda M_{nod,j}$, $\Lambda M_{hp,j}$, $i = 1, 2, 3$ for the blades, and $\Lambda M_{nod}$, $\Lambda M_{yaw}$ for the shaft.

A significant reduction of the damage equivalent load on the blade roots and shaft cross-sectional bending loads can be expected from a well tuned individual pitch control working in turbulence (Bottasso et al., 2013; Riboldi, 2012). Furthermore, the value of shaft cross-sectional bending moments averaged in time should decrease with respect to the value obtained from the baseline trimmer when individual pitch control is activated (Bossanyi, 2003b; Bottasso et al., 2013; Riboldi, 2012). Consequently, the quantities in equation (10) are also of interest for the turbulent wind scenario. Similarly, the definition of actuator duty cycle in equation (11) adopted for the ideal case also applies to this scenario.

As with the ideal case, it is advisable to study normalized values of the quantities of interest instead of their actual values, to assess more directly whether the performance index is increasing or decreasing with respect to the baseline. Summarizing, the following set of performance indices can be considered in the case of the turbulent wind scenario

$$
\rho(\Delta M_{nod}) = \frac{[\Delta M_{nod}]_{IPC}}{[\Delta M_{nod}]_{ref}} \\
\rho(\Delta M_{hp}) = \frac{[\Delta M_{hp}]_{IPC}}{[\Delta M_{hp}]_{ref}} \\
\rho(\Delta M_{yaw}) = \frac{[\Delta M_{yaw}]_{IPC}}{[\Delta M_{yaw}]_{ref}} \\
\rho(\Delta M_{ref}) = \frac{[\Delta M_{ref}]_{IPC}}{[\Delta M_{ref}]_{ref}} \\
\rho(\Delta M_{nod}) = \frac{[\Delta M_{nod}]_{IPC}}{[\Delta M_{nod}]_{ref}} \\
\rho(\Delta M_{hp}) = \frac{[\Delta M_{hp}]_{IPC}}{[\Delta M_{hp}]_{ref}} \\
\rho(\Delta M_{yaw}) = \frac{[\Delta M_{yaw}]_{IPC}}{[\Delta M_{yaw}]_{ref}} \\
\rho(\Delta ADC) = \frac{[\Delta ADC]_{IPC}}{[\Delta ADC]_{ref}}
$$

(15)

where all quantities are ratioed with respect to a condition where the trimmer is operating alone, and $[\Delta ADC]_{ref}$ is defined as in equation (13).
To measure the effect of individual pitch control in turbulent conditions, considering an array of multiple $N_{U,t}$ time histories of wind with different average speed values, a merit function as in equation (16) can be formulated, based on the quantities in equation 15). This yields

$$J_{t} = \frac{1}{N_{U,t}} \sum_{j=1}^{N_{U,t}} \left( p_{j} \rho(M_{\text{Out},j}) + q_{j} \rho(M_{\text{Orig},j}) \right)$$

$$+ r_{j} \rho(M_{\text{nod},j}) + s_{j} \rho(M_{\text{yaw},j})$$

$$+ t_{j} \rho(M_{\text{nod},j}) + m_{j} \rho(M_{\text{yaw},j}) + a_{j} \rho(\text{ADC}_{j}) \right)$$

where $p_{j}$, $q_{j}$, $r_{j}$, $s_{j}$, $l_{j}$, $m_{j}$ and $a_{j}$ are positive scalar weightings. As with $J_{c}$, all components in $J_{t}$ are positive, so that the merit function is definite positive. A good performance of the individual pitch control control law should bring all load indices below unity, at the price of an increase in $\rho(\text{ADC}_{j})$, which will be above unity. Furthermore, as was the case for $J_{c}$, a more intense control action should lower the load indices while at the same time tending to increase pitch motion. Hence, a minimum for $J_{t}$ is likely to appear for a proper selection of the tuning parameters.

Management of weightings

The weightings in equations (14) and (16) contribute to the shaping of the merit functions. In particular, the ratio between the weightings attributed to the measures of load and the actuator duty cycle can be chosen to make the positive effect of the former or the detrimental effect of the latter prevail in the overall value of the considered merit function.

Good practices for setting the values of the weightings in merit functions in engineering problems are largely dependent on the specific problem itself, and usually involve some arbitrary choices. In this case, two considerations led to the set-up of guidelines for choosing the weightings.

Firstly, what is known from the industry is that the effect of the actuator duty cycle is reflected in major technical problems leading to premature consumption of actuators, and making major faults more likely, in turn reducing energy production over time, owing to an increased time required for machine servicing. This finally leads to an increase in costs. The attitude towards individual pitch control shown by the industry is consequently that of considering an increase in the actuator duty cycle as a particularly worrying effect of this control logic. Conversely, as can be seen from many quantitative results in the literature (Bottasso et al., 2014; Stol, 2003; Stol and Balas, 2003), the normalized increase in the actuator duty cycle can be more substantial than the normalized reduction of load indices; hence, even with neutral (unitary) weightings, the attitude of the chosen merit functions is expected to feature a greater sensitivity to actuator duty cycle than to the other measures of performance. For this reason, lower values of $a_{j}$ with respect to other weightings should be imposed in the computation of $J_{c}$ and $J_{t}$.

Secondly, turbine certification standards (IEC, 2005) promote the use of a Weibull distribution for analyzing performance indices depending on the wind speed. The effect of the adoption of the Weibull distribution in the computation of $J_{c}$ and $J_{t}$ has been tested by setting the weightings in the merit functions in equations (14) and (16) to different values for the considered average wind speeds, scaling a starting value assigned by the user for each simulation according to a Weibull probability distribution. In analytical terms

$$w_{j} = w_{j}^{*} \cdot W(U_{0j}(t)), \quad j = 1, \ldots, N_{U}$$

where $w_{j}$ is the actual value of the weighting, $w_{j}^{*}$ is the generic initial weighting value, not accounting for the effect of the Weibull scaling (possibly equal for all wind speeds), $W(U_{0j})$ is the Weibull function for the assigned certification class of the machine and $N_{U}$ is the number of considered wind speeds.

Results

All results were obtained working on the FAST (Jonkman and Buhl, 2005) model of an existing onshore three-bladed, horizontal-axis, upwind 3.0 MW turbine. The flexible modes of blades, shaft and tower allowed by FAST have been enabled, providing for a sufficiently accurate analysis of the response of the machine in terms of deformation and loads. The model was preliminarily validated with respect to the more sophisticated multibody/finite element method code Cp-Lambda (Bauchau et al., 2003; Bottasso and Croce, 2012). The FAST model has been coupled with an external controller, in which the trimmer and individual pitch control laws have been implemented together with supervisory features for managing shut-on, shut-off and emergency procedures (Riboldi, 2012).
For this analysis, a suitably tuned linear quadratic regulator was used to trim the machine in all operating regions (Bottasso et al., 2012; Riboldi, 2012). Being the innermost control layer, the trimmer is tasked with the regulation of power output, computing the torque and collective pitch inputs based on basic machine measures, including the speed of the rotor (Bottasso et al., 2012, 2013; Riboldi, 2012). To cope with the varying quality of the various wind speed regions of the operating envelope, a reduced model of the system was linearized for several wind speeds between cut-in and cut-out, thus implementing a scheduling of the trimmer with respect to the wind. The minimum pitch value considered for trimming is slightly higher than the lower hard pitch limit of the machine, allowing the individual pitch control signal, added to the collective control signal of equation (6), to be actuated mostly free of hardware constraints, even for low wind speeds in the partial power region.

As this analysis is centered on the behavior of pitch control, the aero-elastic model of the wind turbine has been augmented with detailed second-order models of the dynamics of pitch actuators. This assures a good fidelity to real field conditions, which is particularly relevant when testing the controller in turbulence.

The analysis of the merit functions in equations (14) and (16) has been carried out through multiple simulation runs of the FAST model of the considered turbine in constant and turbulent wind, respectively, with different settings of the individual pitch control tuning parameters. The FAST simulation code was deemed sufficiently accurate for the proposed analysis, mainly centered on relative changes in blade and shaft loads; it was also selected because of the need to launch a relatively large amount of simulations while containing the overall execution time of the simulation runs. As a matter of fact, the mapping of the merit functions may be comparatively computationally heavier than optimization in terms of execution time, so great care should be taken to contain the overall length of the mapping process, including selection of a lighter simulator.

The opportunity to check the merit functions visually is of great importance for understanding the shape that they assume. This can be guaranteed in principle only when simultaneously studying the effect of at most two parameters. For this reason, to favor the ease of interpretation of the analysis results, it was decided to study the effect on the merit functions of separate pairs of parameters.

The results of the proposed approach to the optimal analysis by sub-problems will be compared with the outcome of more comprehensive optimization runs on more than two parameters together in a later subsection.

The first pair of parameters are the proportional and integral gains $k_p$ and $k_i$. The second pair are $R_c$ and $R_o$, defining the ranges of activation of individual pitch control. The effect of the first pair has been considered on both $J_x$ and $J_t$; that of the second only on $J_t$.

The reason for this, besides saving on the computational time and length of the study, is the different aim of the study for the two proposed scenarios, and their respective merit functions. The ideal scenario, thanks to the relatively reduced simulation time and ease of analysis of the results, is that most frequently considered for manual tuning of the individual pitch control gains, whatever the implementation (Bossanyi, 2004; Riboldi, 2012; Stol, 2003). Furthermore, in the ideal scenario, the constant value of the characteristic multiples of the rev forcing frequency over time makes it easier to verify theoretical principles from control theory. Conversely, moving to a more realistic turbulent scenario enables verification of the system in slightly off-design conditions, a step closer to the operating conditions found in real environment. By comparing the effect of gains in the ideal and turbulent wind scenarios and their respective merit functions, some conclusions can be drawn about their relevance to the tuning procedure.

The need to implement conditional activation arises from practical considerations bound to the excessive load on actuators and the reduction of the advantages provided by individual pitch control in the field, i.e. far from ideal constant wind conditions (Bottasso et al., 2014). For this reason, it makes sense to check the effect of the characteristic ranges $R_c$ and $R_o$ mainly in turbulent wind conditions and on $J_t$, to determine whether this technological solution improves the performance of individual pitch control in more realistic conditions.

**Effects of individual pitch control gains**

**Constant wind scenario.** For the constant wind scenario, for every set of assigned control design parameters, $N_{u_e} = 12$ simulations of 100 s in constant wind have been carried out. The wind speeds adopted for these simulations are from the cut-in $U_{c, i} = 3$ m/s to the cut-out $U_{c, o} = 25$ m/s, at intervals of 2 m/s.

A square test grid was considered, based on values of the proportional and integral gains of individual pitch control changing in the interval $[-4\times 10^{-8}, 0]$ rad/(Nm) at intervals of $2\times 10^{-9}$ rad/(Nm) for the proportional gain $k_p$, and $[-4\times 10^{-8}, 0]$ rad/(Nm s) at intervals of $2\times 10^{-9}$ rad/(Nm s) for the integral gain $k_i$. The individual pitch control gain attenuation factor $\zeta$ was set to 0.5 between $U_{0} = 10$ m/s and $U_{0} = 12$ m/s, i.e. in an interval close around the rated speed of $U_{c, o} = 11$ m/s. These settings were deemed reasonable after visually checking the time histories of some cases with the proposed boundary values for the gains, measuring the corresponding peak-to-peak amplitude of the pitch signal and
monitoring the behavior of torque, rotational speed of the rotor and power output for unacceptably aggressive control performance.

As highlighted in a previous section, individual pitch control receives Coleman-transformed measures of blade root bending moments as inputs. Blade moments are preliminarily low-pass filtered with a fourth-order Butterworth low-pass filter, limiting the bandwidth of the signals to the 1P frequency, to avoid spurious contribution from higher harmonics to the transformed loads.

A first analysis is concerned about the effect of weightings. It is interesting to check what effect can be obtained with the incremental addition of the various terms in $J_c$. This can be studied by starting with all-zero weightings, then incrementally setting them to non-zero values.

The plots in Figure 1 were obtained by computing the value of $J_c$ for null weightings on all quantities except $g_j = h_j = 10$ (a), $l_j = m_j = 10$ (b) and $a_j = 0.1$ (c), respectively. Figure 1(d) was obtained by superimposing the previous three plots, i.e. by considering a merit function where the weightings on blade load peak-to-peak are $g_j = h_j = 10$, those on shaft load average $l_j = m_j = 10$, and that on the actuator duty cycle $a_j = 0.1$. For all these cases, the Weibull scaling effect was not included in the weightings, so all weightings are the same for all considered wind speeds. The quantity presented in the plots is the relative change of the merit function $J_c$ with respect to the value $J_{c,ref}$ obtained for $k_i = k_p = 0$, or in analytical terms

![Figure 1](image.png)

**Figure 1.** Contour plots of $\delta J_c$ as a function of $k_e$ and $k_o$ (no Weibull scaling). (a) Null weightings except $g_j = h_j = 10$. (b) Null weightings except $l_j = m_j = 10$. (c) Null weightings except $a_j = 0.1$. (d) Weightings $g_j = h_j = l_j = m_j = 10$, $a_j = 0.1$. 

\[ \delta J_c = \frac{J_c - J_{c, \text{ref}}}{J_{c, \text{ref}}} \]  

By constitution, the quantity \( \delta J_c \) assumes a null value for null individual pitch control gains, i.e. in the reference condition when only the trimmer is controlling the machine. A negative value of \( \delta J_c \) corresponds to a better performance with respect to the reference, whereas a poorer performance with respect to the reference would result in a positive value of \( \delta J_c \). Consequently, for better clarity, positive values of \( \delta J_c \) corresponding to an uninteresting performance of the controller are not reported in the plots. The only exception is Figure 1(c), where \( \delta J_c \) assumes only positive values because only the actuator duty cycle has a non-null weighting for that case, and this parameter increases proportionally to the intensities of individual pitch control gains.

A good balance between the components of \( J_c \) is confirmed by Figure 1(d), where it is possible to recognize the effect of the three components of \( J_c \) (peak-to-peak of blade moments, average of shaft moments and average actuator duty cycle) selectively weighted in plots (a), (b) and (c).

In particular, the position of the minimum for higher values of the proportional gain \( k_p \) in Figure 1(d) reflects the effect seen in Figure 1(a), and bound to the top reduction of the peak-to-peak of blade loads obtained for a small range of \( k_p \) and a precise value of \( k_i \). The effects on \( J_c \) of the average shaft loads seen in Figure 1(b) and of the actuator duty cycle presented in Figure 1(c) are clearly opposed, and together influence the shape of the merit function that can be observed in Figure 1(d). They create a gradient in the direction of \( k_i \) that from the top of Figure 1(d), i.e. for null \( k_i \), is initially sharply negative for increasing absolute values of \( k_i \) then becomes mildly positive after reaching the minimum of the merit function, up to a condition where all advantage is lost, owing to an excessively aggressive control action, and \( \delta J_c \) returns to zero.

The areas of Figure 1(d) close to the bottom-left and bottom-right corners, while again corresponding to negative values of \( \delta J_c \), are not of any practical interest. The relatively low values of \( J_c \) in such regions are dictated by a lower actuator duty cycle component, as can be seen in Figure 1(c), in turn because in some simulations individual pitch control is so aggressive that it is deactivated by the control supervisory system, to avoid disturbances to the trimming action and negative consequences to the actuators. In such conditions, only the collective component regulated by the trimmer remains active, thus the corresponding motion of the actuators is limited, causing a lower actuator duty cycle.

Considering the plot in Figure 1(d), it can be observed that for an assigned proportional gain \( k_p \) the merit function is gently increasing for a range of values of the integral gain \( k_i \) beyond the optimum. However, it is also noteworthy that a very intense negative gradient shows up in the \( k_i \) direction for very small values of \( k_i \) (close to the top of the plot), indicating that a good performance advantage can be obtained with a very modest, albeit non-null, contribution of the integral control component.

The effect of the proportional gain \( k_p \) on the merit function is generally more limited than that of the integral gain. For a null value of the integral gain \( k_i \) (i.e. at the top of the plot), it is possible to observe a negative gradient of \( J_c \) for increasing absolute values of the proportional gain \( k_p \), but this gradient is less intense than the aforementioned one in the \( k_i \) direction for an assigned \( k_p \). Notwithstanding the generally more reduced effect of proportional control, the position of the minimum value of \( J_c \) in the case represented in Figure 1(d) is obtained for a non-null value of \( k_p \), showing that the contribution of this control component is not negligible, as far as the actual optimal solution is pursued.

As previously indicated, it should be remarked that an appropriate tuning of the weightings should make the merit function depend on all components in such a way as to possibly balance their effects, without any of them overwhelming the others. In particular, as previously reported, the actuator duty cycle increases rapidly with higher absolute values of the gains, and increasing \( a_j \) to a greater extent than shown in Figure 1, leaving the other weightings unaltered, would result in a merit function totally dominated by the rapid increase of the actuator duty cycle performance index. This, in turn, would make this merit function less suitable for an optimal analysis.

To provide a more complete picture of the sensitivity of the merit function with respect to the weightings, some results obtained by changing the weightings in \( J_c \) are presented in Figure 2. Values for the weightings considered in Figure 2 are variations from the reference conditions \( g_c = h_j = M = 10 \) and \( a_j = 0.1 \). The latter set of weightings is the one previously used for the Figure 1(d), with which the plots in Figure 2 should be compared.

In particular, the plots on the top row refer to weightings \( g_c = h_j = 5 \) to the left and \( g_c = h_j = 20 \) to the right, respectively, leaving all other weightings unaltered. Similarly, in the middle row, results obtained with weightings \( I_c = M = 5 \) to the left and \( I_c = M = 20 \) to the right are reported, whereas on the bottom row, \( a_j = 0.05 \) and \( a_j = 0.2 \) have been considered for the plots to the left and to the right, respectively.

From Figure 2 it can be observed that, despite an expected intense change in the values assumed by \( \delta J_c \) in the various proposed cases, the general shape of the merit function tends to remain basically unchanged, so the remarks presented for
Figure 2. Contour plots of $\delta J_c$ as a function of $k_P$ and $k_I$, showing the effects of changes to the weightings (no Weibull scaling). 
(a, b) Changes in $g_j$ and $h_j$. (c, d) Changes in $l_j$ and $m_j$. (e, f) Changes in $a_j$. 
the case for Figure 1(d) concerning the gradients and the position of the minimum still apply when halving or doubling the weightings considered for Figure 1(d).

In greater depth, by comparing the results of Figure 2(a) and (b) with the plot in Figure 1(d), it can be observed that by upscaling the weightings on blade load peak-to-peak, as for Figure 2(b), the minimum tends to be located in a slightly more pronounced dip in the merit function, whereas by reducing the same weightings, the merit function tends to be flatter around the optimum (Figure 2(a)). Figure 2(a) and (b) are indeed very similar, as witnessed by the limited change in the optimal level of $\delta J_c$, indicating a mild impact of the weightings of the load peak-to-peak $G_j$ and $h_j$ on the general shape of the merit function, all other weightings being equal.

Conversely, Figure 2(c) and (e) bear a larger difference with respect to the corresponding plots, Figure 2(d) and (f). By inspecting these four plots more accurately and recalling that positive values of $\delta J_c$ have not been included, it can be observed that Figure 2(c) and 2(f), obtained by reducing the weightings on the shaft average loads and increasing the weighting on the actuator duty cycle, respectively, are clearly similar in shape, both featuring an evident region of optimality for a relatively broad array of values of $k_e$ and a very limited range of values of $k_i$. Also Figure 2(d) and 2(e), obtained by increasing the weightings on the shaft loads and lowering the weighting on the actuator duty cycle, respectively, are very similar and feature a less pronounced gradient in the $k_i$ direction, and a generally wider region of near optimality of the merit function.

Notwithstanding their different shapes, the positions of the minima in all plots in Figure 2 do not change much from one another, especially in terms of the integral gain $k_i$, and for a range of $k_e$ values roughly between one-third and three-quarters of the top negative gain considered.

To summarize, an analysis referring to an ideal constant wind testing scenario shows that, provided the weightings are suitably tuned, the selected merit function $J_c$ has a shape such that it can be optimized with respect to the individual pitch control gains $k_e$ and $k_i$. Furthermore, it is immediately evident that the integral action of the controller is mostly influential when limited but non-null values of the corresponding gain $k_i$ are assumed. However, although a certain proportional action is necessary for obtaining the actual minimum of the merit function, the action of the proportional controller is comparatively less pronounced than that of the integral term in reducing $J_c$.

**Turbulent wind scenario.** For the turbulent wind scenario $N_{ua} = 12$ simulations in IEC Category A (IEC, 2005), wind with average speeds from $U_{h, a} = 3$ m/s to $U_{r, a} = 25$ m/s, at intervals of 2 m/s, with a time length of 600 s were performed. The same range was considered for $k_e$ and $k_i$ as for the ideal constant wind case. In addition, the settings of the gain attenuation factor $\zeta$ from the previous scenario were maintained.

The weightings in $J_i$ were balanced following considerations similar to those illustrated for the constant wind scenario, yielding the set of weightings $p_{e} = q_{e} = r_{e} = s_{e} = 20$, $l_{e} = m_{e} = 1$ and $a_{e} = 0.5$. Interestingly, compared with the weightings obtained for balancing the effects of the loads and the actuator duty cycle on $J_c$ in the constant wind scenario, the weightings on the damage equivalent load of the shaft and blade loads need to be larger than those on the peak-to-peak of the blade loads in the constant wind case. This suggests that the effect of individual pitch control on the damage equivalent load in turbulence is less marked than that on the peak-to-peak in constant wind. Conversely, the balance between the actuator duty cycle and shaft average loads indicates a generally greater effectiveness of individual pitch control on these loads when compared with the constant wind case.

The results obtained in the turbulent scenario on the merit function $J_i$ are presented in Figure 3. To the left, the weightings $p_{e} = q_{e} = r_{e} = s_{e} = 20$, $l_{e} = m_{e} = 1$ and $a_{e} = 0.5$ are considered, whereas in the plot to the right the same weightings are used as starting values and multiplied by the appropriate value of the IEC Weibull function (IEC, 2005) for the corresponding average wind speeds as in equation (17), yielding different weightings for simulations at different wind speeds. The quantity $\delta J_{\alpha}$, formally defined in the same way as $\delta J_c$ in equation (18), by replacing $J_c$ with $J_i$, is considered in both plots in Figure 3. As with $\delta J_c$, only negative values of $\delta J_i$ are considered for the plot, positive values being representative of an uninteresting, detrimental control performance.

From Figure 3(a), it can be observed that the result on $J_i$ in turbulent wind is not much different from that in Figure 1(d) in terms of general shape, of the direction and intensities of the gradients and of the location of the minimum. This very relevant fact stands in support of a gain tuning procedure based only on the use of less computationally heavy constant wind simulations, without involving the more complicated and time-consuming turbulent wind scenario in the analysis. Like the constant wind scenario, the gradient in the $k_i$ direction is very intensely negative for an assigned intermediate value of $k_e$ and a low absolute value of $k_{\alpha}$ and comparatively mildly positive for increasing absolute values of $k_e$ beyond the optimum. This suggests that the level of $J_i$ can be maintained close to the optimum in terms of $k_i$ for a range of integral gains, but that a non-null value of this gain is necessary to achieve a good performance. As for the constant wind scenario, the gradient in the $k_e$ direction is less marked than in the $k_i$ direction, but the optimum is clearly characterized by a non-null proportional gain, in a range between one-half and three-quarters of the absolute value of the top negative gain considered in the analysis.
Figure 3(b) is based on the same simulation results considered for Figure 3(a) but the weightings for each simulation are scaled with respect to their respective average speed according to equation (17). The effect is mainly that of shifting the position of the near-optimality region, especially increasing the value of $k_I$ corresponding to the optimum. The intensity and location of the gradients in the $k_P$ direction compare well with those of Figure 3(a), as well as the position of the optimum in terms of $k_P$.

It is worth noting that a procedure for optimal gain selection based on the merit function $J_t$ may have different solutions in terms of $k_P$ and $k_I$ in the proposed turbulent scenario, depending on whether the Weibull distribution is taken into account or not, but, owing to the moderated gradient of the merit function in both cases along the directions of both $k_P$ and $k_I$ in a region surrounding the minimum of $J_t$, the difference in the level of optimality for the optimal and a near-optimal combination of the gains is very limited. In other words, an optimal solution for one of the two problems will also be near-optimal for the other. Despite the similarity in the shapes of the two merit functions close to the respective optima in Figure 3(a) and (b), removing the Weibull weighting would be advisable for a preliminary assessment of control performance, whereas, for an evaluation of the overall performance of the controller over the lifespan of the turbine, such scaling should be accounted for. In this fashion, comparing Figure 3(a) and (b), it is possible to argue that a more intense value of $k_I$, which, as previously indicated, appears to be necessary for keeping the control solution to the optimum when considering the Weibull scaling, plays an important role when considering the lifespan of the turbine, whereas a more limited but non-null $k_I$ together with a proportional action would suffice when not accounting for long-term effects.

As a closing consideration, it can be noted that the results obtained in this section are largely in accordance with the theory of individual pitch control, for which the integral control component has a primary relevance in the performance of this control layer, whereas the effect of proportional control should, in principle, be unnecessary. Such a theory is developed for an ideal system in strictly time-periodic conditions, but the results presented here for more realistic constant and turbulent wind scenarios, working on a sophisticated model of a real machine, tend to confirm its predictions (Bossanyi, 2004; Bottasso et al., 2013; Riboldi, 2012; Van Engelen and Kanev, 2009).

**Effect of conditional activation**

An alternative to the use of individual pitch control over the full span of wind speeds between cut-in and cut-out is proposed in Bottasso et al. (2014), showing potentially interesting results on the envelope of the machine when selectively activating individual pitch control only for some wind speeds. In that study, Bottasso et al. devoted much attention to maximizing the effect of individual pitch control on some ultimate load top-scoring in the ranking of loads for the considered machine. The selection of the activation range in the scope of that analysis was basically a matter of hand tuning.

In the current analysis, the merit function $J_t$ provides a measure of the performance of individual pitch control, comprising both the advantageous effects on loads and the detrimental effects on the actuator duty cycle; hence, the effect of the amplitude of the activation range can be studied through analysis of $J_t$. 

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**Figure 3.** Contour plot of $\delta J_t$ as a function of $k_P$ and $k_I$. (a) No Weibull scaling of weightings. (b) All weightings scaled with Weibull function based on the corresponding average wind speed. Non-null weightings on time average of shaft loads, damage equivalent load of blade and shaft loads, and actuator duty cycle.
As for the case of gains, one of the objectives of the present analysis is that of favoring a visual perception of the shape of the merit function. For this reason, to study the effects of conditional activation on $J_t$, only the two amplitudes $R_r$ and $R_o$ of the activation ranges have been considered as tuning parameters, respectively centered at the rated wind speed minus 2 m/s, $U_{0,2} = 9$ m/s, and cut-out wind speed $U_{0,o} = 25$ m/s. The choice of the values of the focal speeds was driven by the already mentioned analysis in Bottasso et al. (2014), and is bound to the relevance of these particular speeds on the load envelope and ranking of a typical three-bladed horizontal-axis wind turbine of multi-MW size. The rated minus 2 m/s wind speed was preferred to the rated wind speed, to avoid overlapping of the activation ranges, thus easing analysis of the results without altering the meaning of the theoretical effects of the parameters. The gains of individual pitch control have been frozen at the values representing the optimal condition according to the analysis in the turbulent scenario proposed previously in this section, visually presented in Figure 3(b), with $k_P = 2.72 \times 10^{-8}$ rad/(Nm) and $k_I = 1.8 \times 10^{-8}$ rad/(Nm $\cdot$ s) for proportional and integral gains, respectively. The setting of the attenuation factor $\zeta$ has been left unaltered with respect to the previous scenarios. The values of range considered in the analysis are in [0, 16] m/s at intervals of 2 m/s for both $R_r$ and $R_o$. Recalling equation (7), it can be noticed that with this choice of values the two operational regions never overlap.

As explained in a previous section, the control logic for activating and deactivating the individual pitch control when entering or leaving an operational region is also responsible for smoothing the transition between the on and off conditions of this control layer. The activation and deactivation procedure is triggered based on a moving-average filtered value of the wind speed measured on top of the tower, the filter having a characteristic time of 10 s. The transition maneuver for individual pitch control activation or deactivation takes a time of 5 s to complete.

Figure 4 shows the behavior of $\delta J_t$ with the same weightings as used for the results in Figure 3 ($p_j = q_j = r_j = s_j = 20$, $l_j = m_j = 1$ and $a_j = 0.5$), when considering the Weibull scaling (b) or not (a).

Considering both plots, it is immediately apparent that the effect of the cut-out range $R_o$ is more visible when the Weibull scaling is not accounted for (Figure 4(a)). This is in accordance with the great effectiveness of individual pitch control for higher values of the wind speed, which is reflected in relevant advantages on loads, in accordance with the literature (Bottasso et al., 2013; Riboldi, 2012). The Weibull scaling significantly reduces the importance of the high wind speed region, hence lowering the effect of parameter $R_o$ with respect to $R_r$.

From both plots in Figure 4, it can be observed that the merit function is lower for higher values of both ranges, with the optimum reached for top values of both $R_r$ and $R_o$. The actual optimal solution in the scenario considered for Figure 4 lies very close to the top bounds of $R_r$ and $R_o$, corresponding to the activation of individual pitch control over all the operating regions of the machine.

However, looking more carefully at Figure 4(b), it can be observed that the level of $J_t$ changes only slightly in a rectangular region extending from $R_r = 14$ m/s to $R_r = 16$ m/s, and for all values of $R_o$. The difference between the maximum and minimum values of $\delta J_t$ in this region is $\delta J_t(14, 0) - \delta J_t(16, 16) = -0.0115$, where the value of $\delta J_t$
corresponding to the optimum is $\delta J_t^{opt} = \delta J_t(16, 16) = -0.0612$. The ratio between the two is 18.8%, representing the maximum performance improvement that can be obtained in the selected rectangular area. For comparison, the value of $\delta J_t$ for $R_r = 14$ m/s and $R_o = 0$ m/s (on the lower boundary of the plot), corresponding to the difference between $\delta J_t(14, 0)$ and $\delta J_t(0, 0)$ (recall the definition of $\delta J_t$ for which when $R_r = R_o = 0$ m/s then $\delta J_t = 0$, i.e. individual pitch control is not working), is $\delta J_t(14, 0) = 0.0497$, and the ratio of this value over $\delta J_t^{opt}$ is correspondingly 81.2%. This represents the maximum improvement that can be obtained with an individual pitch control action only around $U_{r_1} = 2$ with an operative range of $R_r = 14$ m/s. It can be observed that the latter difference is by far more significant than the former, confirming the small relevance of $R_o$ in optimizing the performance.

The results in Figure 4 clearly show that testing the effect of gains considering the Weibull scaling or not may alter the tuning procedure of the control system, making some design parameters almost irrelevant. As previously stated, although not including the Weibull scaling may be acceptable for a preliminary analysis, this is not the case for a more design-oriented analysis, accounting for the performance of the machine over the intended lifespan. According to this consideration, the scenario corresponding to the right of Figure 4 would be more relevant, and this would, in turn, reduce the importance of $R_r$ in the computation of the optimal tuning condition.

The shape of the merit function remains basically similar to that shown in Figure 4, also altering the values assumed for $k_p$ and $k_i$ in the optimal region shown in Figure 3(b). However, altering the weightings in $J_t$ may produce some changes to the merit function. As an example, in Figure 5(a) and (b), the map of $\delta J_t$ with respect to $R_r$ and $R_o$ is presented for two sets of Weibull-scaled weightings, where the starting value of the weightings is the same as for Figure 4, except for $a_j$, which has been obtained by starting from doubled and halved $a_j^* = 1$ and $a_j^* = 0.25$, respectively. The same optimal values of the gains considered for Figure 4 were also considered here.

From Figure 5, it can be seen that when doubling the weighting on the actuator duty cycle (Figure 5(a)), a region of local minimum appears in the shape of the merit function around $R_r = 6$ m/s and $R_o = 16$ m/s. The absolute optimum remains close to the top values of $R_r$ and $R_o$, i.e. for the individual pitch control working at every wind speed. Considering $R_r = 6$ m/s and $R_o = 0$ m/s, the gain with respect to what is obtained with an only collective controller ($R_r = R_o = 0$ m/s) is 45.8% of the optimum $\delta J_t^{opt}$. This result shows that a higher weighting on the actuator duty cycle, meaning a greater relevance of actuator strain in the computation of the overall performance, generates a scenario where roughly half of the global performance is obtained by using individual pitch control only in a limited region close around the rated wind speed.

Conversely, the result obtained by reducing the weighting on the actuator duty cycle (Figure 5(b)) is very similar to that previously shown in Figure 4(b), which was the result of the same balance between all terms in $J_t$, considered when studying the effects of gains in the previous section.
**Optimization results**

As previously explained, the mapping of the merit function was also carried out to provide a visual understanding of the behavior of $J_\text{c}$ and $J_\text{i}$ with respect to the control design parameters. For this reason, the effect of two parameters at a time has been considered.

The results presented suggest a greater influence of $k_\text{i}$ with respect to $k_\text{p}$, considering $J_\text{c}$ in an ideal wind scenario and $J_\text{i}$ in a turbulent wind scenario, respectively, whereas $R_\text{p}$ is more influential than $R_\text{c}$ on $J_\text{c}$ in turbulent wind, especially when considering the Weibull distribution for scaling the weightings. The latter scenario is probably that of greatest interest from the viewpoint of the industry, owing to the greater realism of turbulent simulations with respect to constant wind simulations, and to the fact that the Weibull probability distribution is usually accounted for in the design process whenever results scattered over the wind speeds for which the machine is designed are taken into account (IEC, 2005).

Considering the turbulent scenario and $J_\text{i}$, in the previous sections, optimal solutions have been explored visually for two separate optimization problems, where the dependence of the merit function has been considered on the parameter pairs $k_\text{p}$, $k_\text{i}$ and subsequently $R_\text{p}$ and $R_\text{c}$.

In this section, some results from numerical optimization runs are presented. There are two main motivations for performing such further analyses: firstly, to assess the correctness and level of precision of the results presented for the two optimal problems just mentioned; secondly, to quantify the difference between the optimal solutions found from such partial analyses and more complete optimization problems where all control design parameters are considered together.

When more than two optimization variables are considered simultaneously, it is not possible to visually check the behavior of the merit function, thus making considerations like those presented in this paper more difficult. However, a more complete optimization problem based on more than two parameters will probably produce a solution in terms of gains and ranges that is different from that obtained by solving the two aforementioned optimizations separately. The only way to check the consistency of the optimal solutions in all such cases is by numerically optimizing the merit function and comparing results.

As highlighted through the results previously presented, the shape of $J_\text{i}$ with an appropriate choice of weightings is suitable for numerical optimization. Moreover, as the merit function appears to be a regular function of the two considered sets of parameters (two gains and two activation ranges), the respective optimizations can be safely run with a state-of-the-art sequential quadratic programming gradient-based algorithm. For the optimization problems for which the shape of $J_\text{i}$ cannot be visually investigated a priori, a genetic algorithm has been used, to reduce the likelihood of selecting a solution corresponding to a potential local minimum. In that case, a population of 20 individuals has been considered, compliant with the specified optimization bounds.

For all problems, the convergence tolerance on the merit function for the numerical optimization method has been set to $10^{-4}$. The optimization problems are unconstrained, with bounds on the optimization variables. Whenever considered, $k_\text{p}$ and $k_\text{i}$ are bound to the interval $[-4 \times 10^5, 0]$ rad/(Nm) and rad/(Nm-s), respectively, whereas $R_\text{p}$ and $R_\text{c}$ are bound to the interval $[0, 16]$ m/s. The focal speeds for the activation range have been specified similarly to the previous sections as $U_{0.2} = 9$ m/s and $U_{0.5} = 25$ m/s, respectively, for $R_\text{p}$ and $R_\text{c}$.

The adopted cost function $J_\text{i}$ was computed based on starting weightings $p_0 = q_0 = r_0 = s_0 = 20$. $l_0 = m_0 = 1$ and $a_0 = 0.5$. These have been scaled according to the Weibull probability function depending on the average wind speed (IEC, 2005). The same $N_{\text{sim}} = 12$ simulations of 600 s in Category A turbulence proposed in the previous sections was considered.

The results of the optimizations are reported in Table 1. The values specified for the active parameters were computed using a numerical algorithm as a solution of the optimal problem, whereas those associated with non-active parameters were specified as fixed data in the computation of the optimal solution. Cases 1 to 3 refer to optimal problems of the same quality of those treated in the previous sections, where $J_\text{c}$ was analyzed as a function of two parameters at a time. In particular, Case 1 refers to the same condition treated in Figure 3, whereas Cases 2 and 3 refer to that treated in Figure 4. Solutions for these optimization problems were obtained with a gradient-based algorithm. Cases 4 and 5 refer to optimization problems with more than two parameters, and these have been solved with a genetic algorithm as previously explained.

The first two cases in Table 1 confirm the results obtained in the previous subsections. Actually, the optimal values of the gains obtained for Case 1 were used to produce the map in Figure 4. The result of Case 2 shows a negligible difference with respect to Case 1 on the active parameters $R_\text{p}$ and $R_\text{c}$. The limited improvement in the level of optimality that can apparently be obtained by slightly reducing the activation ranges of individual pitch control is probably due to the adopted simulation grid, and running multiple simulations with various turbulence seeds is likely to push the optimal solution towards a condition corresponding to a full-span activation, in accordance with the results presented in Figure 4.

Case 3 was implemented to check the change of the optimality level when $R_\text{p}$ is not considered as a parameter for optimization. Considering Cases 2 and 3 together, it can be seen that a limited percentage increase in $\delta J_\text{opt}$ of about 17.1% is obtained by inhibiting individual pitch control around the cut-out, in practice reducing it drastically from $R_\text{c} = 15.7$ m/s
to $R_o = 0$ m/s. This shows that, as preventively assessed in the previous analyses, $R_o$ is not a critically relevant control design parameter as far as Weibull scaling is considered, being responsible for less than one-fifth of the overall performance improvement.

Generally speaking, the results of Cases 1–3 tend to confirm the validity of the observations on the results of the previous sections (Figures 3 and 4).

From the analysis of the results of Cases 4 and 5, it is apparent that a numerical optimization simultaneously accounting for more than two parameters does not provide significant advantages in terms of change to the optimal value of $J_{t\text{opt}}$ with respect to the corresponding sub-problems. The values of the optimal gains are not much different from those obtained for Case 1. Furthermore, considering the most general case, Case 5, the values of the ranges are not much different from those obtained for Case 2, whereas the value for the optimal $R_o$ in Case 4 is close to that found in the similar case, Case 3. Once again, comparing the value of $\delta J_{t\text{opt}}$ for Cases 4 and 5, a loss of performance $\delta J_{t\text{opt}}$ of about 16.5% is obtained when excluding parameters $R_o$ from the optimization problem. As discussed for Cases 2 and 3, this means that by reducing $R_o = 15.7$ m/s to $R_o = 0$ m/s, and slightly adjusting the other free optimization parameters, a relatively small loss in optimality is obtained, limited to under one-fifth of the best performance, hence confirming a light effect of $R_o$ in the considered optimization problem.

Comparing the results in Table 1, it appears that the solution of the optimization problem over four parameters can be split into two separate sub-problems without significant losses in the level of optimality, i.e. in the value $J_{t\text{opt}}$ and consequently $\delta J_{t\text{opt}}$. This further justifies the approach by sub-problems proposed in this paper, which is not only advantageous for obtaining a visual understanding of the behavior of the merit function and of the significance of the various terms in it, but also enables an effective search for an optimal condition that is very close to the actual one, found by solving a more complete optimization problem.

This notion can be effectively exploited to reduce the computational effort required for tuning. All gradient-based optimizations (Cases 1–3) were completed with a convergence tolerance of $10^{-4}$ on $J_t$ with a number of evaluations of the merit functions in the range 30–50. Genetic algorithms applied to this optimization problem (Cases 4 and 5) with the same tolerance ($10^{-4}$) on the difference between the scores of individuals and a population of 20 individuals for each generation can easily require 700–900 evaluations. The latter figures should be taken with caution, for, as this research demonstrated, the actual position of the optimum for $J_t$ as a function of more than two parameters does not fall far from the respective optima obtained for partial problems based on two parameters; hence, the initial population has been selected accurately close to the most likely position of the optimum, thus possibly easing convergence. Conversely, even though the position of the actual optimum of the merit function is not altered by a change in the weightings, some form of dependence on their values does exist in the considered scenario, as shown, for instance, in Figure 5. Further changes to the weightings, possibly due to particular design necessities aimed at penalizing some terms most, may alter the shape of the function more markedly, potentially causing the need to increase the size and scatter of the initial population of a genetic algorithm so as to safely avoid local minima. However, where the problem of optimality based on all parameters can be split into two sub-problems, it would be possible to switch safely from a genetic algorithm to a gradient-based one, with a great saving in computational time.

### Conclusions

The focus of the research presented in this paper lay in studying the feasibility of an optimal tuning procedure for an individual pitch control law to be applied to horizontal-axis wind turbines, and consequently of understanding how to set up such a procedure.

### Table 1. Results of optimizations. Active parameters for optimization as specified. Values reported for non-optimized parameters are specified in the optimization routines. Values reported for optimized parameters were computed as optimal values by the optimizer. Optimal values of $\delta J_{t\text{opt}}$ computed accounting for Weibull weighting. Cases 1–3: solution through gradient-based algorithm. Cases 4 and 5: solution through genetic algorithm.

<table>
<thead>
<tr>
<th>Case</th>
<th>$k_p$</th>
<th>$K_i$</th>
<th>$R_o$</th>
<th>$R_o$</th>
<th>$\delta J_{t\text{opt}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Active</td>
<td>Value</td>
<td>Active</td>
<td>Value</td>
<td>Active</td>
</tr>
<tr>
<td>1</td>
<td>Yes</td>
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<td>Yes</td>
<td>$-1.80 \times 10^{-8}$</td>
<td>No</td>
</tr>
<tr>
<td>2</td>
<td>No</td>
<td>$-2.72 \times 10^{-8}$</td>
<td>No</td>
<td>$-1.80 \times 10^{-8}$</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>No</td>
<td>$-2.72 \times 10^{-8}$</td>
<td>No</td>
<td>$-1.80 \times 10^{-8}$</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>Yes</td>
<td>$-2.77 \times 10^{-8}$</td>
<td>Yes</td>
<td>$-2.38 \times 10^{-8}$</td>
<td>Yes</td>
</tr>
<tr>
<td>5</td>
<td>Yes</td>
<td>$-2.73 \times 10^{-8}$</td>
<td>Yes</td>
<td>$-1.83 \times 10^{-8}$</td>
<td>Yes</td>
</tr>
</tbody>
</table>
To this aim, a realistic architecture of an individual pitch control control law was proposed and implemented. The parameters of interest for the tuning process were defined as the proportional and integral gains of the individual pitch control law, as well as the extensions of the two individual pitch control operational ranges centered at two different wind speeds.

Two operational scenarios were considered for studying the performance of the controller for different values of the tuning parameters; an ideal constant wind scenario and a turbulent wind scenario. Performance indices measuring the positive effects on loads and the detrimental effects on pitch actuator strain were defined and assembled in merit functions suitable for an optimality study.

The importance of the weighting process for both the ideal and turbulent wind scenarios was investigated. In particular, it was shown that normalized changes in actuator duty cycle should be accompanied by a relatively small weighting in the merit function, owing to their naturally greater intensity with respect to load performance indices. This is in spite of the great relevance that the actuator duty cycle generally has for industrial subjects. Failing to balance weightings suitably may result in a merit function that does not feature an optimum within the assigned bounds of the optimization parameters. Such bounds have a physical meaning. For the case of gains, they were chosen to avoid an excessively intense or fast action of individual pitch control, resulting in a poor control action, due also to the filtering action of pitch control commands by the dynamics of pitch actuators, and possibly likely to force the control supervisor to intervene in shutting off the turbine. For the case of activation ranges, focal speeds and bounds are designed to cover the whole operating range of the turbine when both range values are at their top limits, while avoiding any overlap.

To obtain a visual analysis of the results, the proposed merit functions were mapped with respect to at most two parameters at a time. The analysis as it was configured with respect to two scalar gains and two amplitudes of the corresponding operational ranges is suitable for a similar mapping.

The dependence of the merit function on individual pitch control gains was investigated, showing that, notwithstanding a certain dependence of the specific optimal gain set on the weightings, the solution tends to be similar for both the ideal and turbulent scenarios, and, for the latter, whether a Weibull distribution is accounted for in the weightings or not. The optimal solution is characterized by a non-null but comparatively small value of the integral gain, and by a value of the proportional gain in a range between one-half and three-quarters of the absolute value of the top negative gain considered for the investigation. A limited shift of the optimum towards a more intense integral gain can be observed when including the Weibull weighting in the turbulent scenario. This is accompanied by less pronounced intensities of the gradients in the directions of the proportional and integral gains around the optimum, implying that near-optimal solutions in a limited but visible area around the optimum are not far from it. This provides a very strong indication that the requirement to consider a more time-consuming and complicated turbulent scenario for tuning the individual pitch control gains is not stringent. Put another way, optimal tuning of the gains may be carried out in the ideal constant wind scenario, the result also being nearly optimal in turbulent wind.

The dependence of the merit function on the operational ranges has been analyzed for the turbulent scenario. Unlike the analysis with respect to individual pitch control gains, the effect of actuator duty cycle weighting is here less pronounced, whereas the choice of including the Weibull scaling in the weightings plays a relevant role. When the Weibull scaling is not considered, the merit function is greatly reduced by increasing the width of the operational range around the cut-out speed, with the size of the range around the rated speed being as important. When the Weibull scaling is accounted for, the roles played by the amplitudes of both ranges are more different, with \( R_{o} \) being almost unimportant. This is in accordance with the fact that, when considering the Weibull scaling, the high speed operational range where substantial advantages on loads are obtained with individual pitch control is less relevant to the global performance.

This demonstrates that accounting for the Weibull scaling or not may imply a simplification in the tuning, as far as these performance indices are considered. If the Weibull function is accepted as reflecting a realistic distribution of the wind speed, or when dictated by industrial needs, in turn stemming from standard requirements, it should be accounted for. If the Weibull function is accounted for, the parameter \( R_{o} \) is not very influential and can be excluded from the tuning procedure.

To check the reliability of the results obtained by considering partial sets of design parameters and to better assess the influence of the various parameters on the quality of the optimal solution, some full numerical optimizations were carried out in the more realistic turbulent scenario. The same Weibull-scaled weightings were considered for all optimizations, whereas different choices of optimization parameters were considered. The results of the optimizations have confirmed that considering different sets of parameters does not alter significantly either the optimal value of the merit function or the corresponding values attributed to the various considered parameters for each case, which are sometimes only slightly different.

The results of the optimizations highlight two important facts concerning tuning. Firstly, the small impact of parameter \( R_{o} \) on the level of the optimal solution in the adopted test scenario is again noted. Secondly, the level of the optima
attained by solving the partial optimizations over two parameters (gains or operational ranges) are very similar to those obtained by solving the optimal problems based on larger parameter sets. This implies that the optimizations of the gains and of the ranges can be carried out as separate processes whenever needed, with a great saving in computational effort and execution time.

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